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ULTIMATE STRENGTH ANALYSIS

A. M. FREUDENTHAL

and

P. Y. WANG

TECHNICAL REPORT AFML-TR-69-60

MARCH 1969

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AIR FORCE MATERIALS LABORATORY
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

This report was prepared by Dr. A. M. Freudenthal, New York, N. Y. under USAF Contract AF 33(615)-3430 with assistance from Mr. P. Y. Wang under contract AF 33(615)2871. The contracts were initiated under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals". Contract AF 33(615)-3430 was administered by Ohio State University Research Foundation, contract AF 33(615)-2871 by Columbia University. The work was monitored by the Metals and Ceramics Division, AF Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp.

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This technical report has been reviewed and is approved.

W. J. Trapp

W. J. TRAPP
Chief, Strength and Dynamics Branch
Metals and Ceramics Division
Air Force Materials Laboratory

ABSTRACT

The results of ultimate static strength tests from different types of aircraft structures and structural parts obtained from several aircraft manufacturers were statistically analyzed. By using test samples with at least 3 replications and reducing sample data to their mean, all results could be unified in a single population of over 300 data points and these points fitted by the Third Asymptotic distribution of smallest values (Weibull distribution). This distribution is used as a representative distribution of the ultimate strength of an aircraft combined with the ratio between the design ultimate load and the ultimate strength attained in actual tests, derived from the test data.

By combining the distribution of strength with representative distributions of gusts in flight through thunderstorm turbulence and of operational loads respectively, realistic reliability functions for ultimate load failure of gust-sensitive (long range) and of maneuver sensitive (short range) aircraft structures were obtained for various assumed levels of the ultimate design load.

This abstract is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division (MAM), Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

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1. Introduction

It is the purpose of this study to estimate the structural reliability of critical parts of airframes on the basis of ultimate strength test data of aircraft structures from various sources and of the spectrum of extreme gust and maneuver loads. The Third Asymptotic distribution function of Extreme (Smallest) Values (Weibull distribution) has been chosen to fit the ultimate strength data obtained from tests on aircraft structural parts. Thus the strength of any one member of the airframe can be expressed in terms of its design ultimate load with the aid of this distribution function.

A comparison is made between the recently obtained test data and the test results obtained by the Air Force about twenty years ago.[1] This comparison throws some light on certain aspects of aircraft structural development during this period. Representative thunderstorm and flight load spectra are adopted to match the distribution function of ultimate strength, and the risk of "ultimate load failure" is computed according to a standard procedure developed for this purpose [2] by assuming various values of the design gust or load factor. The associated reliability functions are subsequently evaluated in terms of the number of load application or of flight time of the aircraft.

2. Analysis of Data

Test data from 19 different types of structures and 38 types of panels have been obtained from various aircraft manufacturers through the efforts of the Air Force Materials Laboratory, AFSC, WPAFB. These data have been analyzed and evaluated according to types of loading, types of structure and number of tests in each group and are summarized in Tables 1 and 2.

The expedient assumption is now made that the distribution of the ultimate strength of the test specimens can be considered to represent a single population, irrespective of the type of structures tested and its mode of failure, as long as this failure can be classified as "ultimate". This assumption unavoidable because replications of ultimate load tests of large structures and structural parts are and will always be severely limited by technical and economic considerations. Without it reliability analysis of aircraft structures becomes obviously impossible since the individual small samples are useless for this purpose.

Many distribution functions have been tried to fit the experimental results, but it appears that the Weibull distribution function provides a reasonably satisfactory representation of the data. The probability of survival expressed in terms of this distribution is given by the expression

$$L_X(x) = \exp \left[- \left(\frac{x}{v}\right)^k \right] \qquad (1)$$

where by definition $L_X(x)$ is the probability of survival $L_X(x) = 1 - P_X(x)$, the variate $X = R_i/\bar{R}_i$, R_i being the ultimate strength of any one member in the i-th group while $\bar{R}_i$ is the group mean of the i-th group; v is the characteristic value of the distribution and k a scale factor.

The values of X ($R_i/\bar{R}_i$) have been calculated for every specimen in each group (at least 3 data values from nominally identical specimens under the same type of loading in each group) and arranged in ascending order of magnitude. These values have been plotted against the plotting position, where m = 1,2...n and n = the total number of data points, on extreme value probability paper as shown in Fig. 1 (data from General Dynamics is not included, see Table 2). In Fig. 1 a straight line has been drawn to fit the data points disregarding extreme points of high strength on the assumption that the required distribution of ultimate strength should be representative towards the low rather than towards the high range of data points. The extreme points at both tails of the distribution have been identified by letters (as listed in Tables 1 and 2).

The parameters of Eq. 1 are obtained graphically:

$$k = 31.0 \quad \text{and} \quad v = 1.014$$

Thus the probability of survival

$$L_X(x) = \exp\left[-\left(\frac{x}{1.014}\right)^{31}\right] \quad (2)$$

When data points from General Dynamics are taken into consideration, the extreme points of low strength deviate considerably from the straight line as shown in Fig. 2. This is simply because the individual results in several groups of General Dynamics data scattered rather widely within the groups, as indicated by the identifying letters in Fig. 2 and Table 2. The reason for this wide scatter is not known; however its existence reveals that a small group of somewhat odd behavior can significantly change the nature of the distribution. The equation of the straight line fitting the data in Fig. 2 is

$$L_X(x) = \exp\left[-\left(\frac{x}{1.017}\right)^{25}\right] \quad (2a)$$

However in the following analysis only Eq. (2) is used.

Some of the experimental data are expressed in terms of "design ultimate load" or of "limit load" (design ultimate load = 1.5 limit load); these groups are identified by asterisks in Tables 1 and 2. As a result the group means of these groups can also be expressed in terms of their design ultimate load. The values of these group means have been plotted against m/(n+1) as shown in Fig. 3. A smooth curve has been chosen to fit these points fairly well, except for one extreme point of high value. The equation of this curve is

$$L_Y(y) = \exp\left[-\left(\frac{y}{0.96}\right)^{24}\right] \quad (3)$$

where $Y = \bar{R}_i/R_{DU_i}$ (R_{DU} = design ultimate load). It is assumed that the inclusion of the test data for which the design ultimate load or limit load has not been specified would not change the form and only insignificantly change the parameters of Eq. (3).

From Eqs. (2) and (3) the distribution and density functions of X and Y are easily obtained

$$F_X(x) = 1 - L_X(x) = 1 - \exp\left[-\left(\frac{x}{1.014}\right)^{31}\right]$$

and (4)

$$F_Y(y) = 1 - L_Y(y) = 1 - \exp\left[-\left(\frac{y}{0.96}\right)^{24}\right]$$

their density functions therefore

$$f_X(x) = \frac{dF_X(x)}{dx} = \frac{31}{1.014}\left(\frac{x}{1.014}\right)^{30} \exp\left[-\left(\frac{x}{1.014}\right)^{31}\right]$$

and (5)

$$f_Y(y) = \frac{dF_Y(y)}{dy} = \frac{24}{0.96}\left(\frac{y}{0.96}\right)^{23} \exp\left[-\left(\frac{y}{0.96}\right)^{24}\right]$$

Since $R_i = X\bar{R}_i$ and $\bar{R}_i = YR_{DU_i}$, it follows that

$$R_i = XY\,R_{DU_i} = ZR_{DU_i} \quad (6)$$

if $Z = XY$. Z is a random variable and R_{DU} the computed design value for a specific critical member of the aircraft structure. The distribution of Z represents therefore the distribution of ultimate strength of critical members of the airframe and thus of the airframes themselves. By definition

$$F_Z(z) = P\{Z \le z\} = P\{XY \le z\}$$

$$= \iint_D f_{XY}(x,y)\,dxdy \qquad (7)$$

$$= \iint_D f_X(x)\,f_Y(y)\,dxdy$$

where D is the domain of integration as shown in Fig. 4; hence

$$F_Z(z) = \int_0^{\infty} f_X(x) \int_0^{z/x} f_Y(y)\,dydx = \int_0^{\infty} f_X(x) F_Y(\frac{z}{x})\,dx$$

or

$$F_Z(z) = 1 - \int_0^{\infty} (\frac{31}{1.014})(\frac{x}{1.014})^{30} \exp\,[-(\frac{x}{1.014})^{31}]\exp[-(\frac{z}{0.96x})^{24}]\,dx \qquad (8)$$

if the expressions of $f_X(x)$ and $F_Y(y)$ are substituted into the above integral.

With the abbreviation $t = \exp[-(\frac{x}{1.014})^{31}]$, Eq. (8) is transformed into

$$F_Z(z) = 1 - \int_0^1 \exp[-(\frac{z}{0.9734})^{24}(-\ln t)^{-24/31}]\,dt \qquad (9)$$

which is a form convenient for numerical evaluation. The associated reliability function

$$L_Z(z) = \int_0^1 \exp[-(\frac{z}{0.9734})^{24}(-\ln t)^{-24/31}]\,dt \qquad (10)$$

is presented in Fig. 5 (solid curve), for all values of Z; this curve can be fitted by the equation

$$L_Z(z) = \exp[-(\frac{z}{0.96})^{19}] \tag{11}$$

The small difference between Eq. (10) and Eq. (11) around Z = 1.0 is shown in Fig. 5.

The mean value and the variance of the random variable Z are[3]

$$EZ = v\Gamma(1 + 1/k)$$

and (12)

$$\mathrm{Var}\ Z = v^2\{\Gamma(1 + 2/k) - [\Gamma(1 + 1/k)]^2\}$$

From Eq. (11) v = 0.96 and k = 19; therefore

$$EZ = 0.933$$

and

$$\sigma_Z = \sqrt{\mathrm{Var}\ Z} = 0.061$$

These values suggest that, in general, the mean value of the ultimate strength of nominally identical members of aircraft structures is about 93 percent of its nominal design ultimate load, while the standard deviation is about 6 percent of the design ultimate load.

All test data expressed in terms of design ultimate load or limit load have also been considered without regard to the requirement of at least three test replications in each group and listed in Table 3. These data have been plotted in Fig. 6 and compared with the results of Jablecki's tests[1] performed during the 1940's. The curve obtained from current data indicates that the processes of design and construction of aircraft structures with respect to ultimate load failure have been improved within the two decades. For example Fig. 6 suggests that about 90 percent of the currently produced aircraft structural members will sustain 80 percent of their nominal design ultimate load without failure, whereas in the 1940's only about 60 percent of the design ultimate load was sustained without failure by 90 percent of the structural members. However, there has been no significant change in the percentage of structural parts that fail to carry about 97 percent of the full nominal design ultimate load: only about one-half of all specimens tested can be expected to sustain this load level without failure.

An attempt has been made to improve the fit of the assumed distribution of X by eliminating the high strength data as shown in Fig. 1 on the assumption that as the third asymptotic distribution of smallest values, it is more representative towards the lower range of data points than in the high range. The new plot is shown in Fig. 7; the equation of the straight line to fit the data points is

$$L_X(x) = \exp\left[- \left(\frac{x}{1.105}\right)^{33.8}\right]$$

with two extreme points of low strength belonging to the same group scattering far from the straight line.

The distribution of Y based on six group means expressed in terms of design ultimate load or limit load has been plotted in Fig. 8 and fitted by the equation

$$L_Y(y) = \exp[-(\frac{y}{0.95})^{24}]$$

On the basis of a similar analysis as discussed in the previous part of this section the probability function is obtained

$$L_Z(z) = \int_0^1 \exp[-(\frac{z}{0.96425})^{24} (-\ln t)^{-24/33.8}]dt$$

and approximated by

$$L_Z(z) = \exp[-(\frac{z}{0.95})^{19}] \qquad (11a)$$

as shown in Fig. 9.

A plot of the data of six groups (35 data points) expressed in terms of design ultimate load or limit load has been presented in Fig. 10. This diagram shows that the data scatter around the straight line of Eq. (11). This fact supports the assumption made in the derivation of $L_Z(z)$ that X and Y are independent random variables.

3. Reliability Analysis For Thunderstorm Turbulence (Long Range Aircraft)

A representative spectrum of thunderstorm gust velocity[4] forms a basis for the distribution function of extreme load intensities selected for the evaluation of the risk of failure and the determination of the reliability function for ultimate strength failure of aircraft structures. In Fig. 11 the probability of exceedance is chosen as ordinate instead of the cumulative frequency of load peaks per mile of flight. The spectrum in Fig. 11 (solid curve) is approximated by the equation

$$\bar{F}_U(u) = 0.22e^{-0.22u} + 0.78e^{-0.20u} \tag{13}$$

where $\bar{F}_U(u) = 1 - F_U(u)$ and U = gust velocity in ft./sec.

For design conditions it is assumed that the gust load is directly proportional to the gust velocity. Hence

$$\frac{\text{gust load}}{R_{DU}} = \frac{\text{gust velocity, U}}{U_{DU}} = \Lambda \tag{14}$$

where U_{DU} is the specific thunderstorm gust velocity corresponding to the design ultimate load, and the ratio Λ, a random variable, represents the variable gust load in terms of the design ultimate load.

From Eq. (13) the distribution function of Λ is

$$F_\Lambda(\lambda) = 1 - \{0.22 \exp(-0.22\ U_{DU}\lambda) + 0.78 \exp(-0.20\ U_{DU}\lambda)\} \tag{15}$$

The reliability function $L_N(N)$ is defined as the probability of survival of the aircraft structures under a series of N load applications, so that

$$L_N(N) = \int_0^\infty [F_\Lambda(z)]^N f_Z(z)dz \tag{16}$$

where $F_\Lambda(z)$ is given by Eq. (14) and $f_Z(z)$ can be obtained from Eq. (11).

For practical purposes the following first approximation of Eq. (16) can be used:

$$L_N(N) = \exp(-N p_f) \quad \text{for} \quad N p_f << 1 \tag{17}$$

where p_f is the probability of failure under single load application obtained from

$$p_f = \int_0^\infty F_Z(\lambda)\, f_\Lambda(\lambda) d\lambda \tag{18}$$

From Eq. (I.11)

$$F_Z(\lambda) = 1 - L_Z(\lambda) = 1 - \exp[-(\frac{\lambda}{0.96})^{19}] \tag{19}$$

and by differentiating Eq. (15) with respect to λ,

$$f_\Lambda(\lambda) = U_{DU}\{0.0484 \exp(-0.22\, U_{DU}\lambda) + 0.156 \exp(-0.20\, U_{DU}\lambda)\} \tag{20}$$

Substituting the above expressions into Eq. (18),

$$p_f = U_{DU} \int_0^\infty \{1 - \exp[-(\frac{\lambda}{0.96})^{19}]\}[0.0484 \exp(-0.22\, U_{DU}\lambda) + 0.156 \exp(-0.20\, U_{DU}\, \lambda)]d\lambda \tag{21}$$

With the substitution $\omega = \exp(-\lambda)$, the above integral is transformed into the form,

$$p_f = U_{DU} \int_0^1 \{1-\exp[-(-\frac{\ell n\, \omega}{0.96})^{19}]\}[0.0484\omega^{(0.22\, U_{DU}-1.0)} + 0.156\omega^{(0.20\, U_{DU}-1.0)}]d\omega \tag{22}$$

which is used for numerical calculation.

For various values of U_{DU}, p_f can be evaluated from Eq. (22), and the reliability function obtained from Eq. (17) as a function of the number of gust load applications. Four values of U_{DU} have been assumed: U_{DU} = 90 ft./sec., 75 ft./sec., 60 ft./sec. and 45 ft./sec.; the corresponding values of p_f and the function $L_N(N)$ are shown in Fig. 12.

The proportion of flight through thunderstorm turbulence is about 0.1 percent of flight distance.[5] It is assumed that 10 gusts per mile are encountered by an aircraft during thunderstorm flight at 10-20,000 ft. level. Assuming further that the design life of the aircraft is 5×10^4 hours and the average flight velocity is 400 miles per hour, the number of load applications is converted into time of flight in hour (4 gusts are equivalent to 1 hour of flight). The reliability function of ultimate strength of aircraft structures can therefore be expressed as a function of flight hours as shown in Fig. 12.

In a recent report[6] the hours to reach or exceed ultimate strength have been computed for two aircraft designed by Lockheed: for the L-188 this figure is 7.14×10^7 hours and for the L-749 2.38×10^8 hours. With these data the reliability functions for these two aircraft have been plotted in Fig. 12. The difference between the present analysis and the Lockheed analysis is probably due to the fact that the latter is apparently involved only with the random character of the applied load while the strength is assumed constant, while in the present analysis the statistical character of strength is combined with that of the gust load. The returns period of failure according to this analysis are 2.42×10^6 hours for $U_{DU} = 90$ ft./sec., 2.02×10^5 hours for $U_{DU} = 75$ ft./sec., 1.52×10^4 hours for $U_{DU} = 60$ ft./sec. and 1.06×10^3 hours for $U_{DU} = 45$ ft./sec.

The values of p_f computed on the basis of Eq. (11a) are 1.20×10^{-7}, 1.42×10^{-6}, 1.84×10^{-5} and 2.54×10^{-4} by assuming $U_{DU} = 90$ ft./sec., 75 ft./sec., 60 ft./sec. and 45 ft./sec. respectively. It can be seen that there is no significant difference between the values of p_f by adopting either Eq. (11) or Eq. (11a) for the ultimate strength distribution of the critical parts of aircraft structures.

4. Reliability Analysis For Operational (Maneuver) Loads (Fighter Aircraft)

A representative spectrum of operational (maneuver) load factors has been constructed on the basis of flight records from the F-105B and F-106A aircraft.[7] In Fig. 13 the probability of exceedance of the load factor n has been plotted for both aircraft and an intermediate spectrum selected for the reliability analysis which can be approximated by the equation

$$\bar{F}_n(n) = 2.935e^{-1.77n} + 13.830e^{-3.32n} \qquad (23)$$

where $\bar{F}_n(n) = 1 - F_n(n)$.

The assumption is now made that, for purposes of design the operational load is proportional to the load factor or

$$\frac{\text{operational load}}{R_{DU}} = \frac{\text{load factor}}{n_{DU}} = \Lambda \qquad (24)$$

where n_{DU} is the design load factor of ultimate load design and the ratio Λ, a random variable, represents the variable maneuver load factor in terms of the design ultimate load.

From Eq. (23) the distribution function of Λ is obtained

$$F_{\Lambda}(\lambda) = 1 - \{2.935e^{-1.77n_{DU}\lambda} + 13.830e^{-3.32n_{DU}\lambda}\} \qquad (25)$$

and therefore

$$f_{\Lambda}(\lambda) = n_{DU}[5.195e^{-1.77n_{DU}\lambda} + 45.916e^{-3.32n_{DU}\lambda}] \qquad (26)$$

The probability of failure according to Eq. (18) with $F_Z(\lambda)$ from Eq. (19)

$$p_f = n_{DU} \int_0^1 \{1 - \exp[-(-\frac{\ell n\ \omega}{0.96})^{19}]\} \cdot [5.195\omega^{(1.77n_{DU}-1)} + 45.916\omega^{(3.32n_{DU}-1)}]d\omega \qquad (27)$$

where $\omega = \exp(-\lambda)$.

Equation (27) has been numerically evaluated for the ultimate design load factors $n_{DU} = 13$, 11, 9 and 7 and the corresponding reliability functions have been constructed in accordance with Eq. (17) and are represented in Fig. 14 in terms of the number of load cycles and of hours of flight. The conversion is based on the assumption that roughly 10^2 load cycles are equivalent to one hour of flight at an average of 350 to 400 mph in accordance with the flight records.

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Table 1. Data On Ultimate Strength Of Structures

Symbol**	Type Of Specimen	Type Of Loading	No. Of Tests
	Typhoon Tailplane: Semi-Span	Bending	14
	Typhoon Tailplane Modified: Semi-Span	Bending	19
	Hudson Tailplane: Semi-Span	Bending	6
S_1	Whitley Tailplane: Semi-Span	Downward Bending	13
	Whitley Tailplane: Semi-Span	Upward Bending	7
S_2	Whitley Tailplane: Semi-Span	Torsion	21
	Mustang Wings: Asymmetric	Bending	5
S_3	F-80 Tailplane	Bending	7
S_4	F-86D Tailplane	Bending	3
	M.I.T. Results { Specimen Type 1 Specimen Type 2	Bending	3 3
	Box-Beam Tests	Bending	9
	*F-51H Wings	Bending	3
	*B-70 Spar-Skin Composite Beam	Compression	3
S_5	*C-130A, C-130B, C-130E Wings	Bending	10
	*C-130A Fuselage Cabin (nose section)	Internal Pressure	4
	*A-26B Wings	Bending	7
	*A3D-2P Wings	Bending	3
S_6	B-58 Sandwich Box Beam	Bending	5
	* Data expressed in terms of DUL or LD. ** For extreme data points only.		145 (19 groups)

Table 2. Data On Ultimate Strength Of Components

Symbol**		Type Of Specimen	Type Of Loading	No. Of Tests
C_1	Convair	Wing Lower Surface Plate Stringer, J and L Type Stiffener	Compression	15
C_2	Convair	Wing Lower Surface Plate Stringer, J Type Stiffener	Compression	7
	Convair	Wing Upper Surface Plate Stringer, Stringer Type "A"	Compression	12
	Convair	Wing Upper Surface Plate Stringer, Stringer Type "B"	Compression	6
	Convair	Wing Upper Surface Plate Stringer, Stringer Type "C"	Compression	6
	Convair	Wing Upper Surface Plate Stringer, Stringer Type "D"	Compression	6
	Douglas	Fuselage Frame Type "A"	Compression	4
	Douglas	Fuselage Frame Type "B"	Compression	4
	Douglas	Fuselage Compression Panel (DC-8)	Compression	8
	Douglas	Wing Compression Panel (DC-8)	Compression Compression Compression	3 3 3
	Douglas	Wing Compression Panel (DC-9)	Compression Compression Compression	3 6 6
	Douglas	Wing Compression Panel (DC-8,Dc-9)	Compression	9
	Douglas	Fuselage Shear Panel (DC-8)	Shear	4
	Douglas	Fuselage Shear Panel (DC-9)	Shear Shear Shear Shear	5 4 3 3

Table 2. Continued

Symbol**		Type Of Specimen	Type Of Loading	No. Of Tests	
C_3	Lockheed*	C-130 Wing Panel	Compression	8	
C_4	Lockheed*	C-130E, C-130B Wing Panel	Compression	8	
	Northrop	YT-38 Vertical Stabilizer Panel	Compression	4	
	Northrop	YT-38 Vertical Stabilizer Panel	Compression	4	
C_5	General Dynamics	Aluminum Inner Skin Panel	Compression	5	54
	General Dynamics	Shear Panel	Shear	5	
C_6	General Dynamics	Beaded Aluminum Panel	Compression	4	
			Compression	4	
C_7			Compression	11	
			Compression	4	
C_8	General Dynamics	Aluminum Beaded Inner Skin Panel	Compression	3	
			Compression	3	
			Compression	3	
C_9	General Dynamics	Sandwich and Beaded Skin Panel	Compression	3	
			Compression	3	
			Compression	3	
C_{10}	General Dynamics	Wing Sandwich Panel	Shear	3	
		* Data expressed in terms of DUL or LD. ** For extreme data points only.		196 (38 groups)	

Table 3. Data On Ultimate Strength Of Structures and Components (in Terms of Design Ultimate Load or Limit Load)

Structures

Type Of Specimen	Type Of Loading	No. Of Tests
*A-26B Wings	Bending	7
*A3D-2P Wings	Bending	3
XA3D-1 Wings	Bending, Torsion	2
XA-2D1 Wing	Bending, Torsion	1
C-124A & C Wing	Bending, Torsion	1
XF-4D-1 Wing	Bending	1
C-133A Wing	Bending	1
A3D-2 Wing	Bending, Torsion	1
*F51II Wings	Bending	3
*C-130A, C-130B, C-130E Wings	Bending	10
*C-130A Fuselage Cabin (nose section)	Internal Pressure	4
C-130A Fuselage Cabin (center section	Internal Pressure	1
C-130A Fuselage Cabin (aft. section	Internal Pressure	1
T-38 Horizontal Stabilizer	Bending	2
*B-70 Spar-Skin Composite Beam	Compression	3

Table 3. Continued

Components

Type Of Specimen	Type Of Loading	No. Of Tests
A3D-2 Front-Spar-Fuselage Frame	Bending, Torsion	1
A3D-2 Front Spar Frame Element	Compression	1
*C-130E, C-130B Wing Panels	Compression	8
C-141A Fuselage Panels	Compression	3
C-141A Fuselage Panels	Shear	5
C-141A Fuselage Panels	Shear & Compression	3
NB-66 Wing Cover Assembly Splices	Compression	4
* Data contained in the plotting of Figs. I-1 and I-2.		66

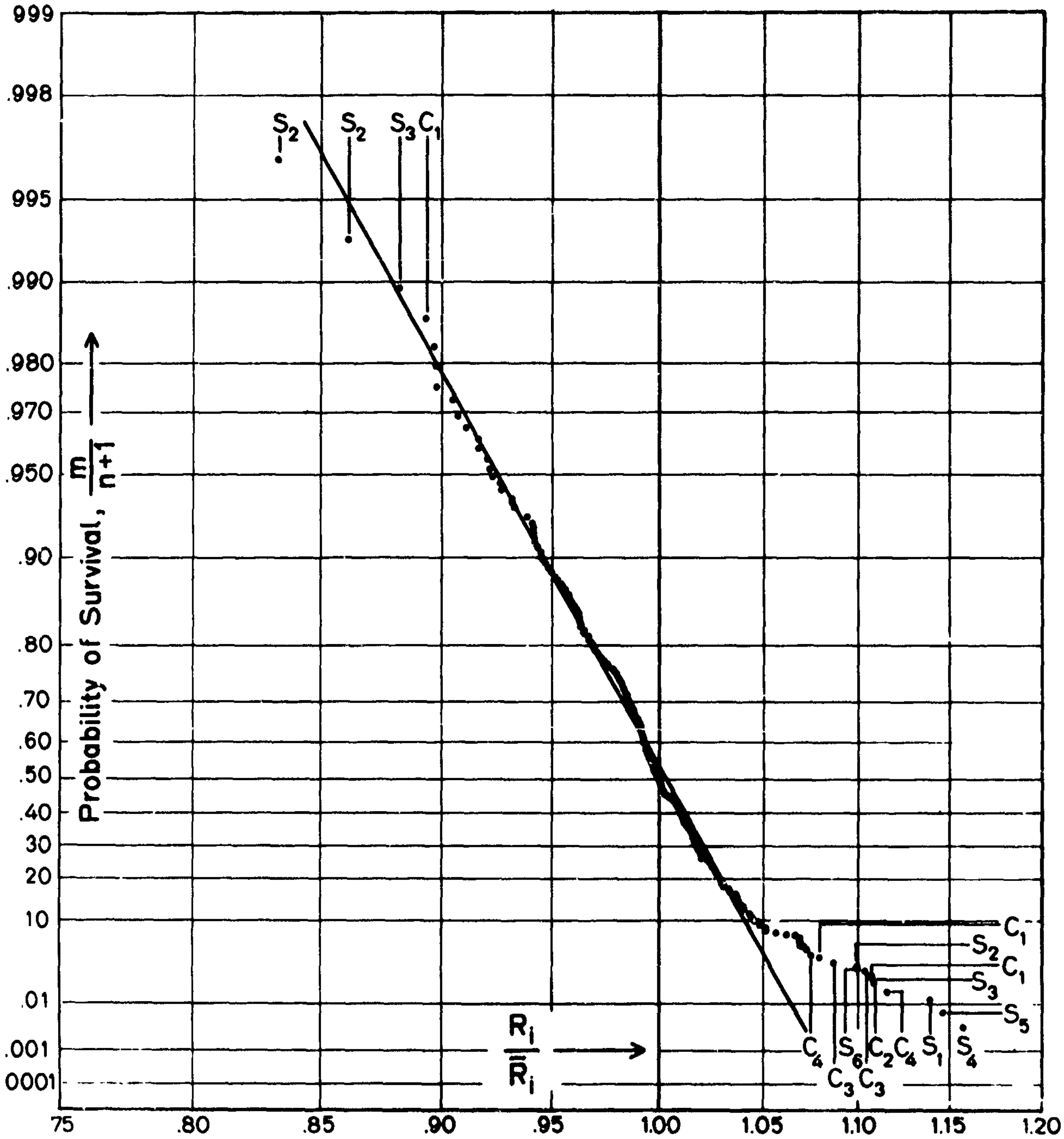

Fig. 1 Probability distribution of structural resistance (287 test data).

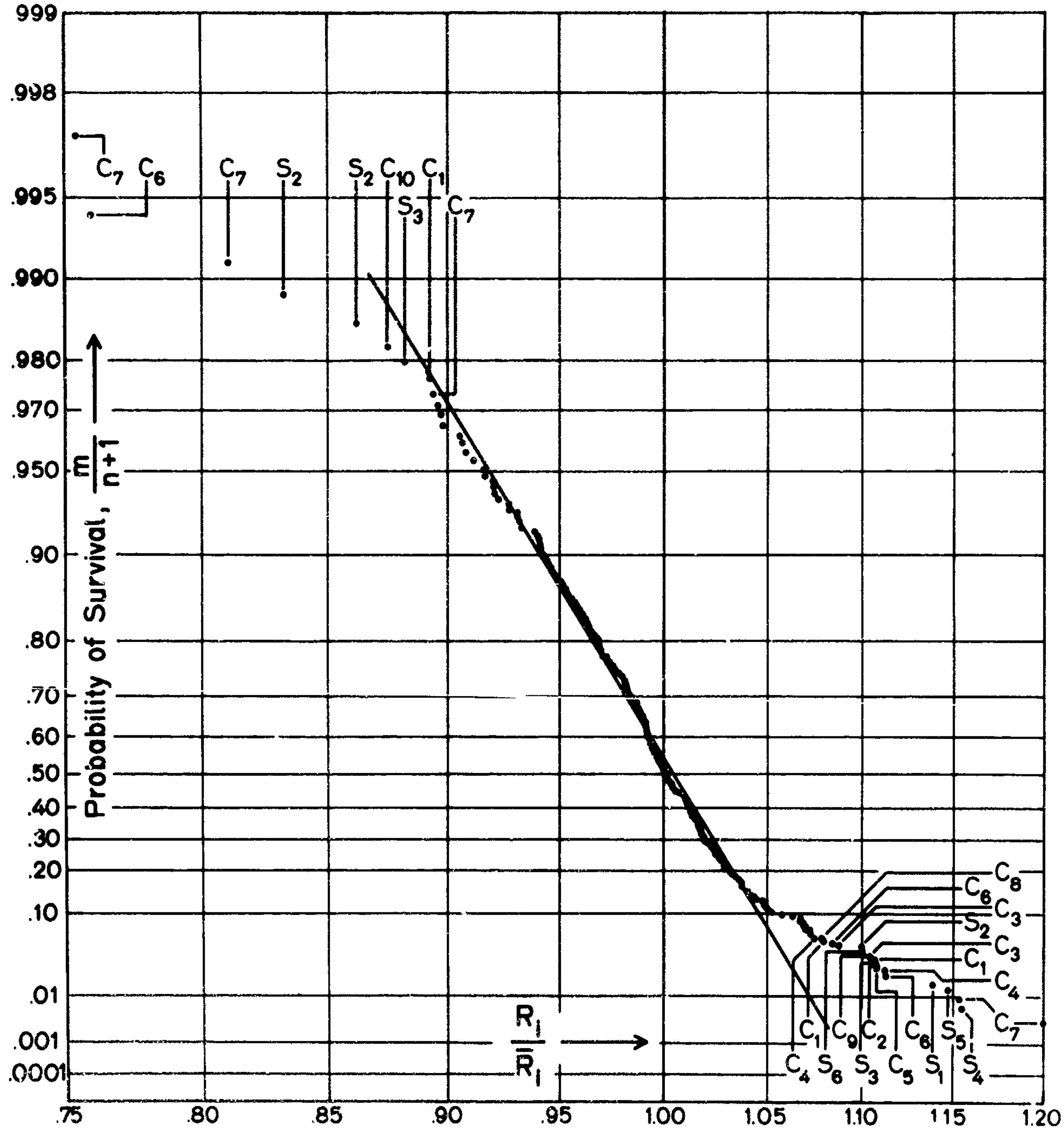

Fig. 2 Probability distribution of structural resistance (341 test data).

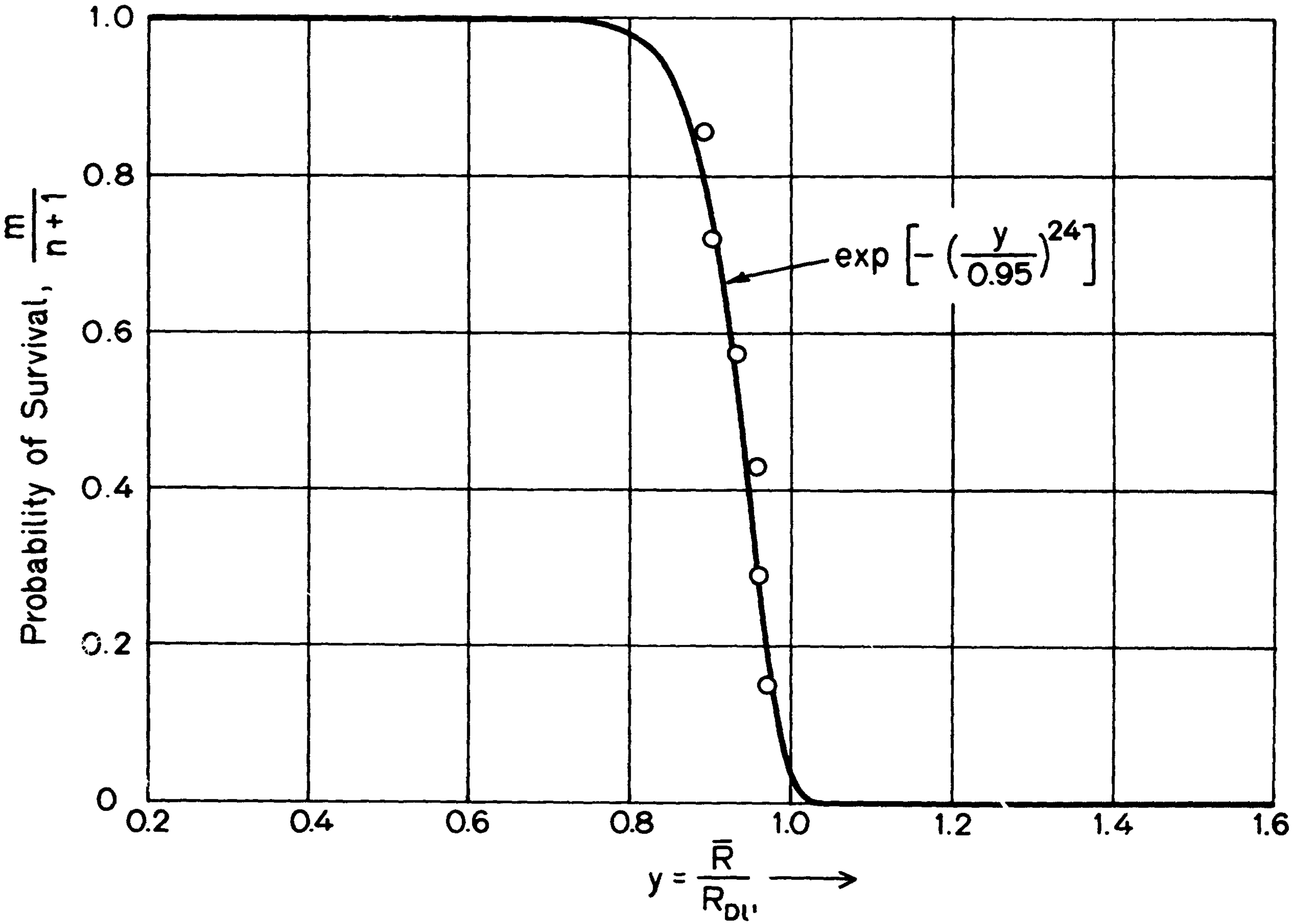

Fig. 3 Probability distribution of group means of structural resistance (based on 7 groups).

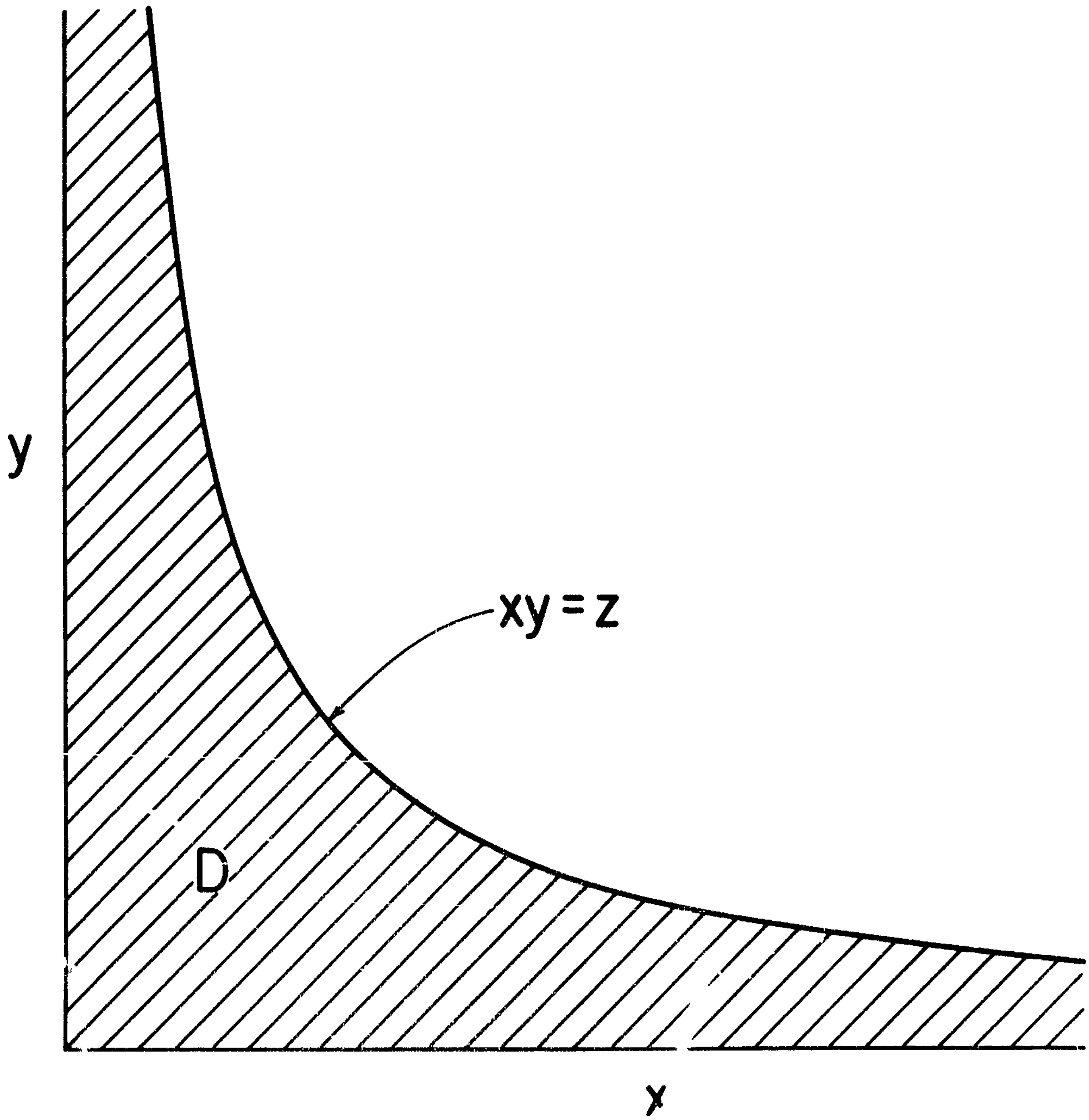

Fig. 4 Domain of integration for the distribution function of Z.

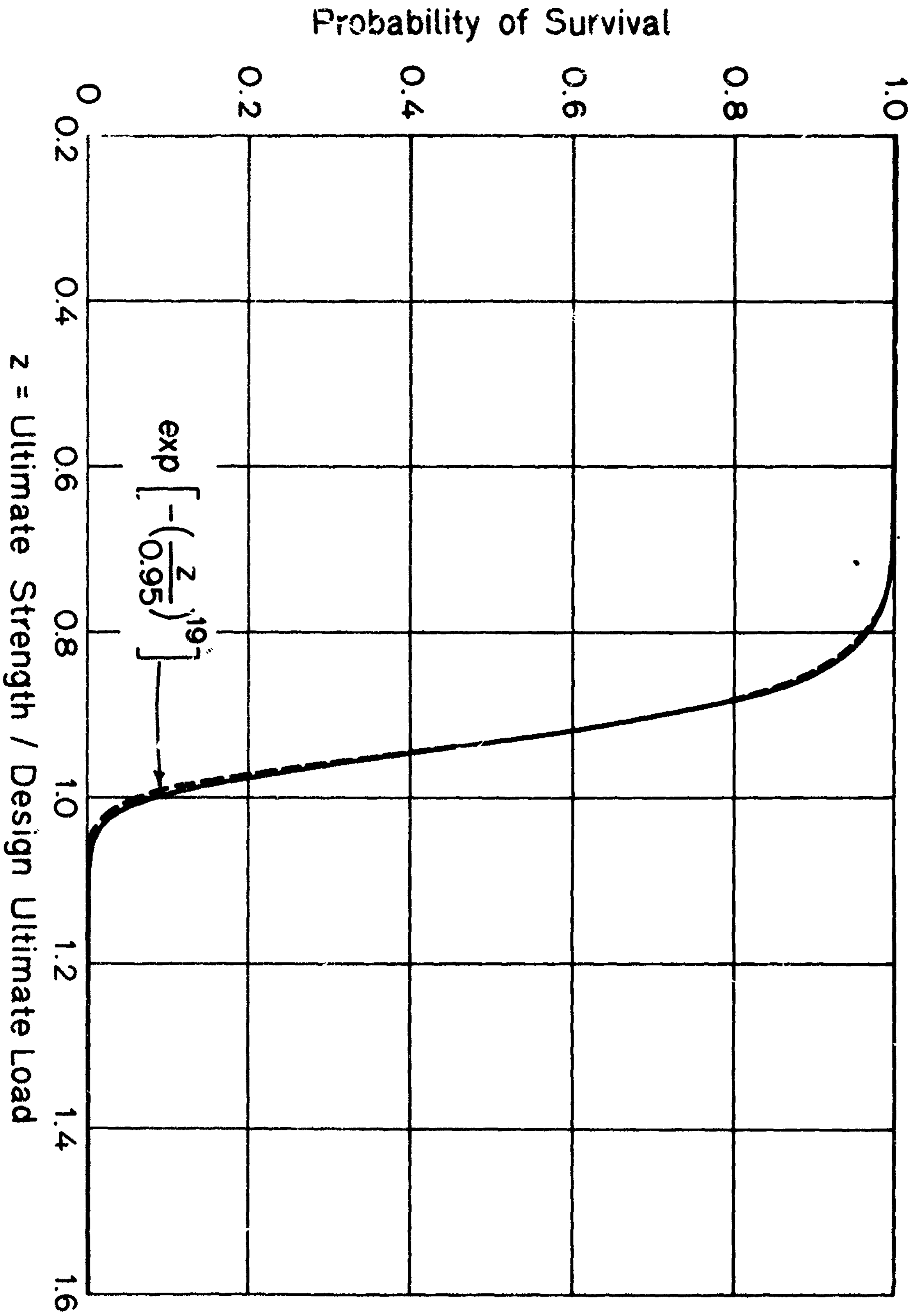

Fig. 5 Probability distribution of Z.

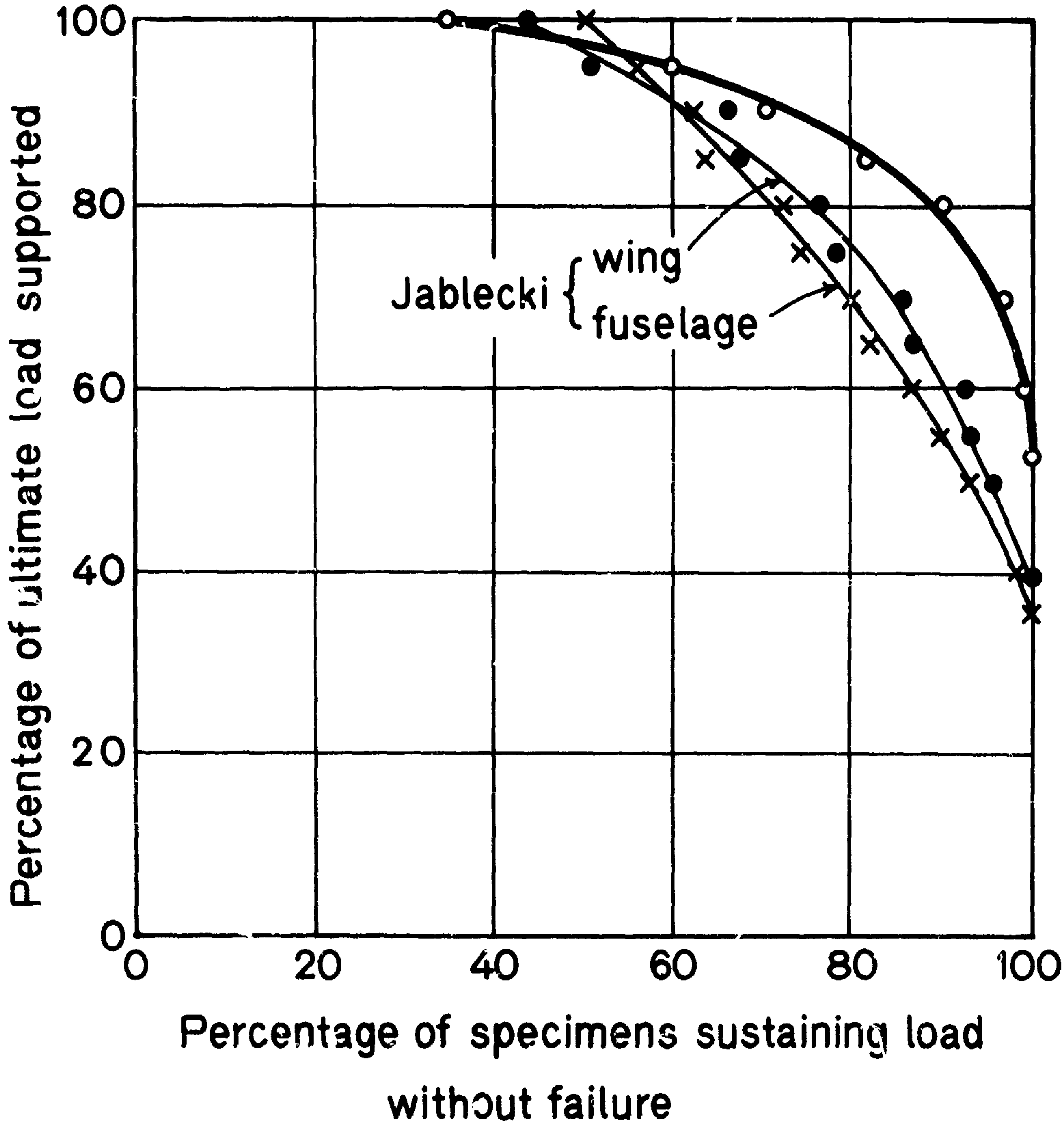

Fig. 6 Test results expressed in terms of design ultimate load (66 tests) compared with results of previous study.[1]

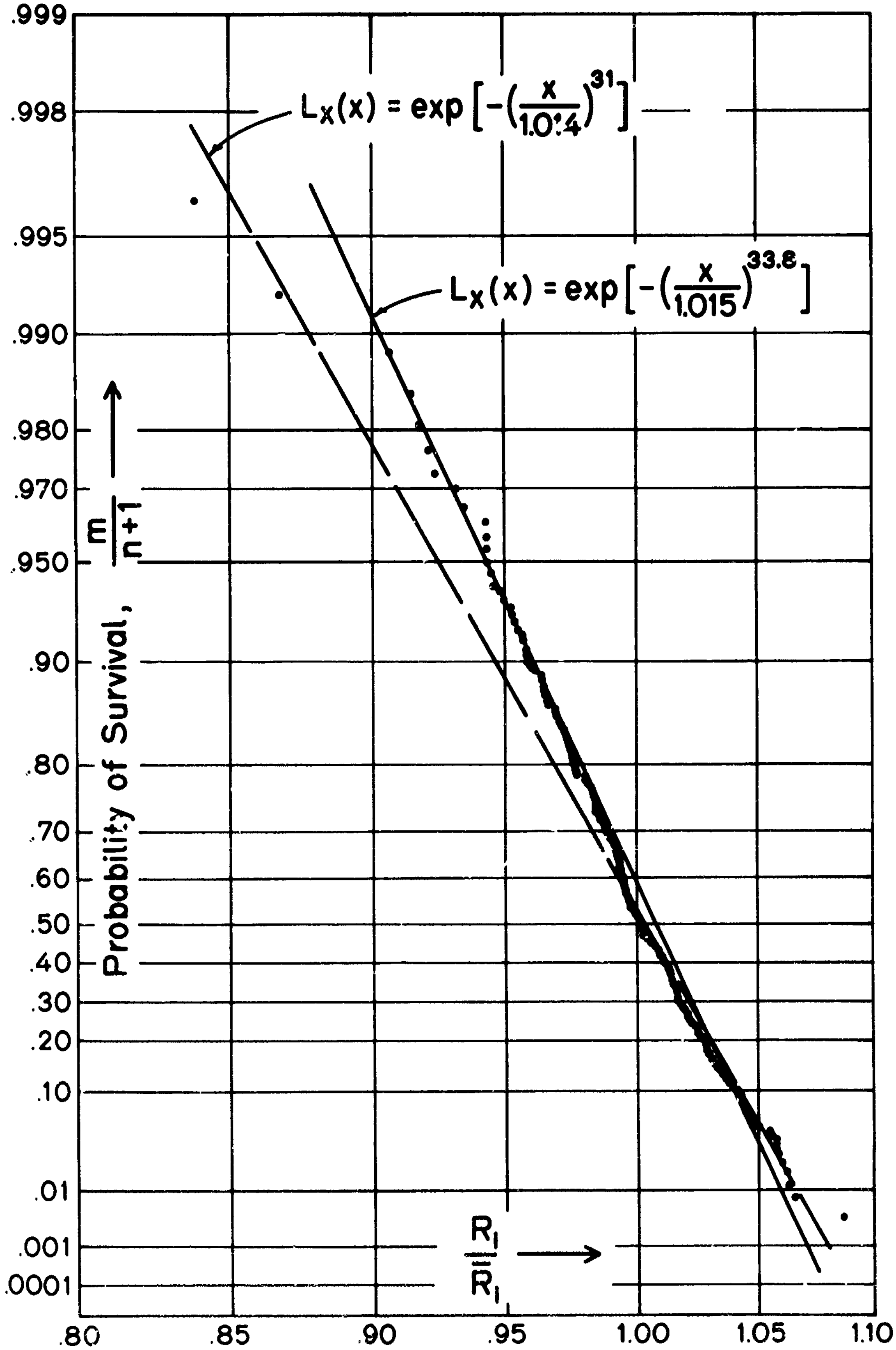

Fig. 7 Probability distribution of structural resistance (258 test data).

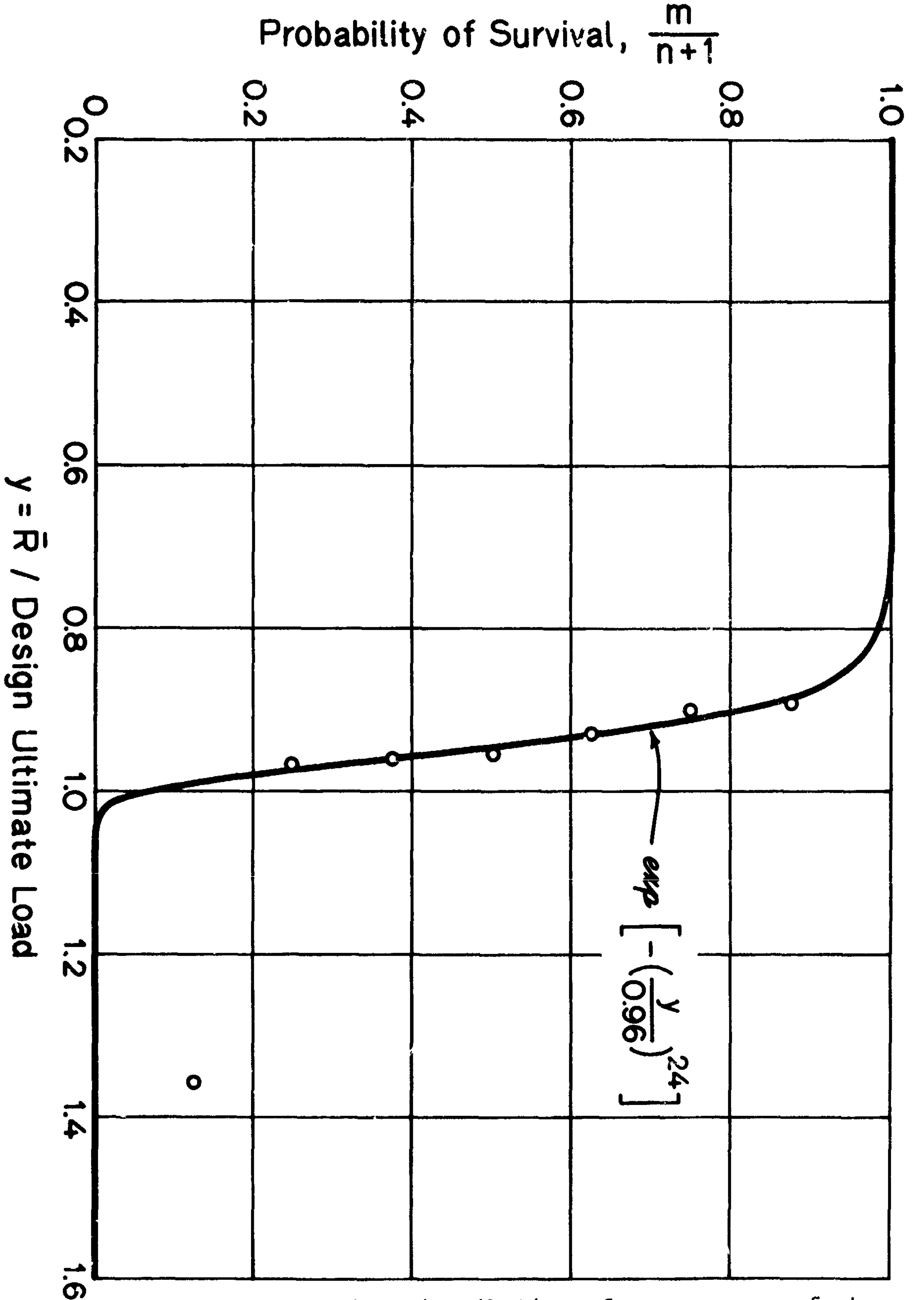

Fig. 8 Probability distribution of group means of structural resistance (based on 6 groups).

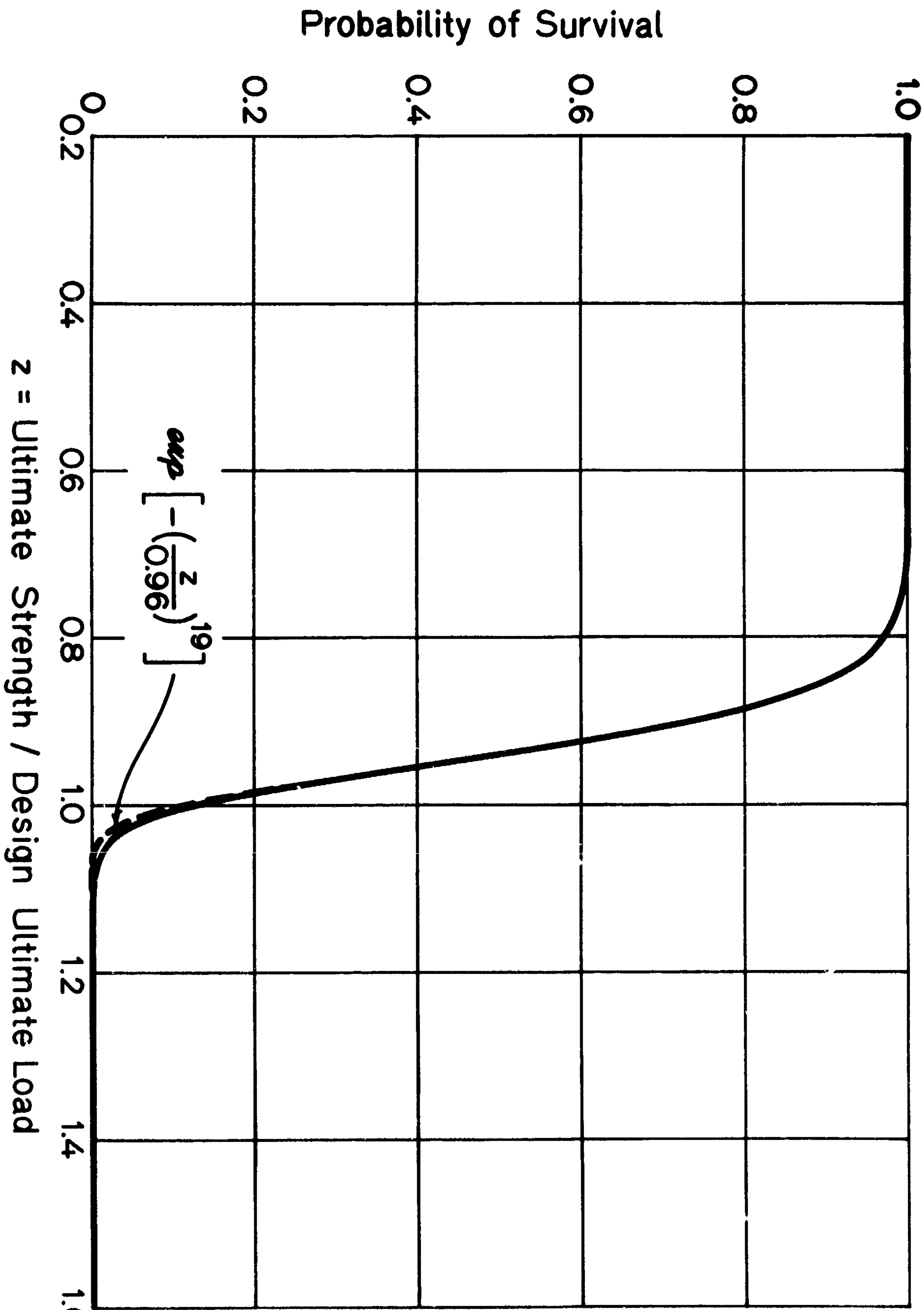

Fig. 9 Probability distribution of Z (without high strength data).

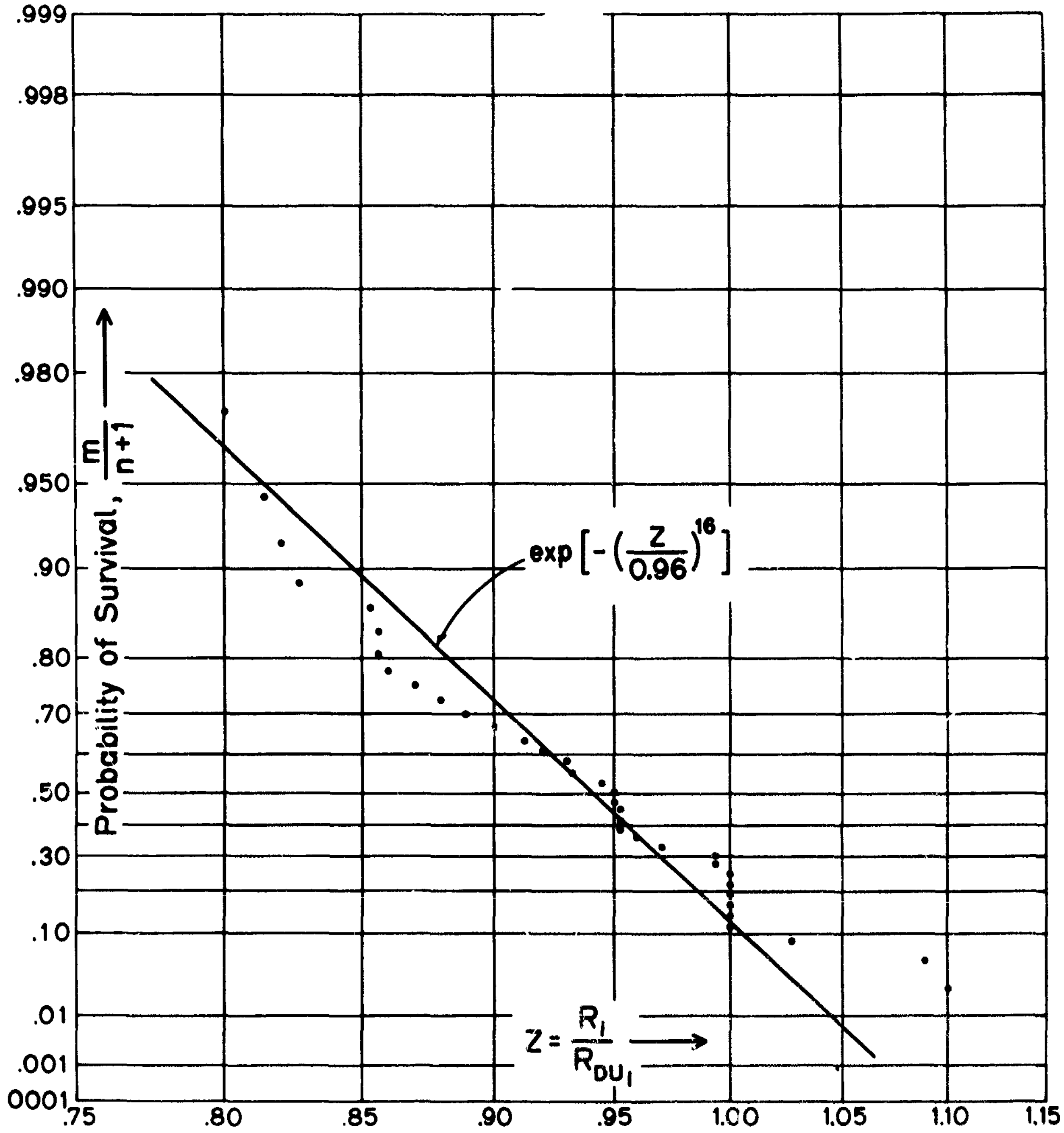

Fig. 10 Probability distribution of structural resistance (based on 35 data points).

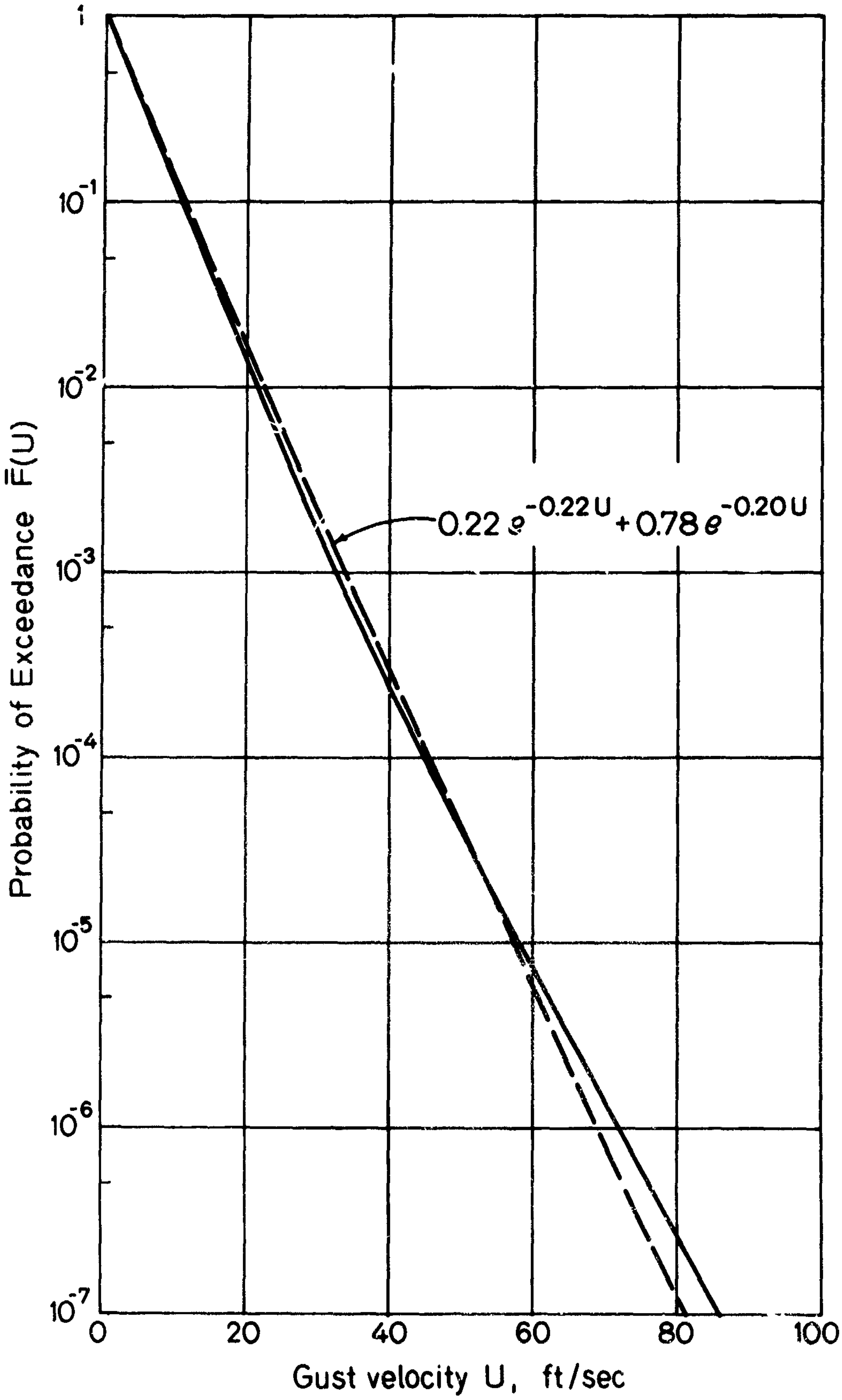

Fig. 11 Probability distribution of thunderstorm gust velocity.

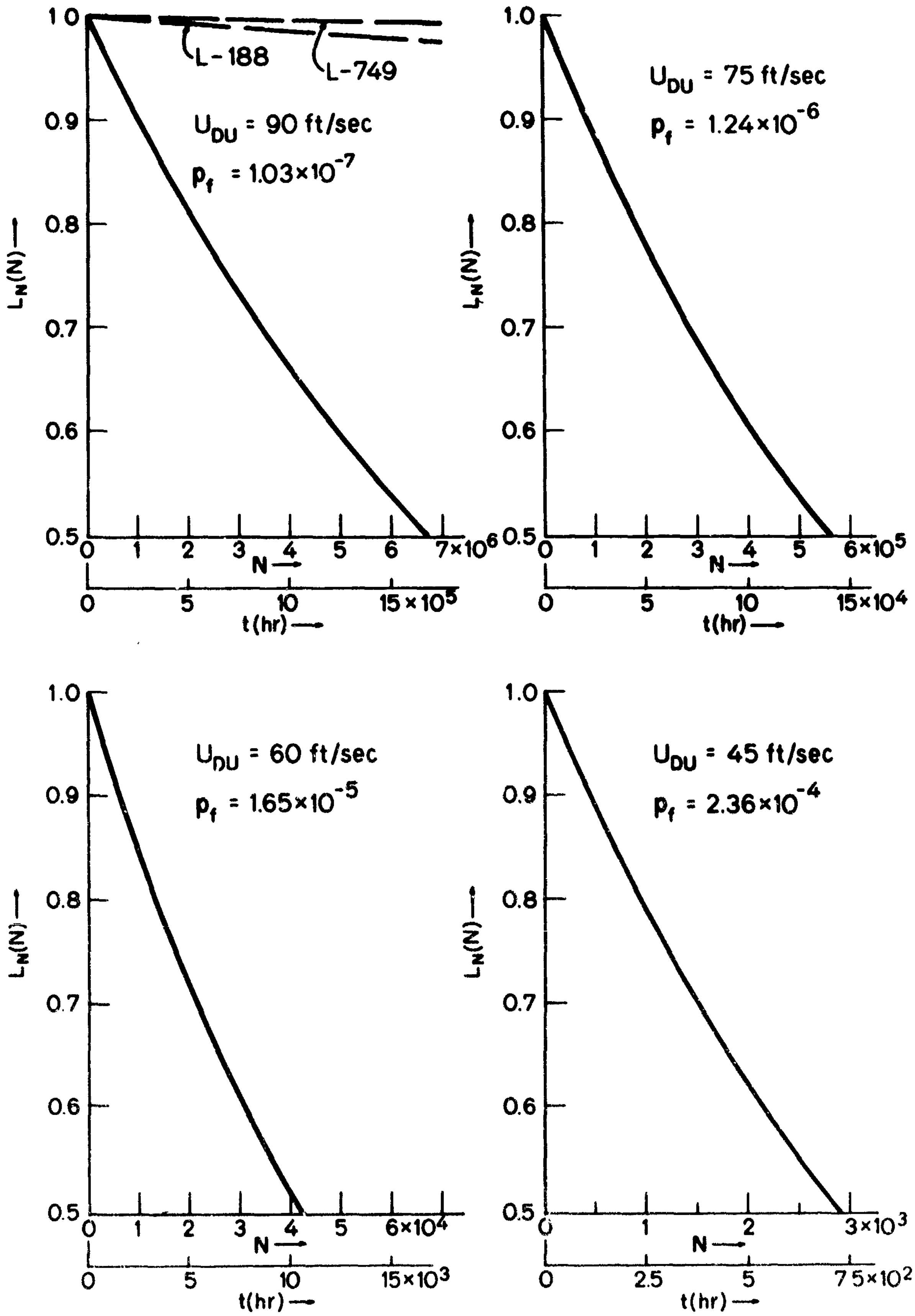

Fig. 12 Probability of survival under thunderstorm turbulence.

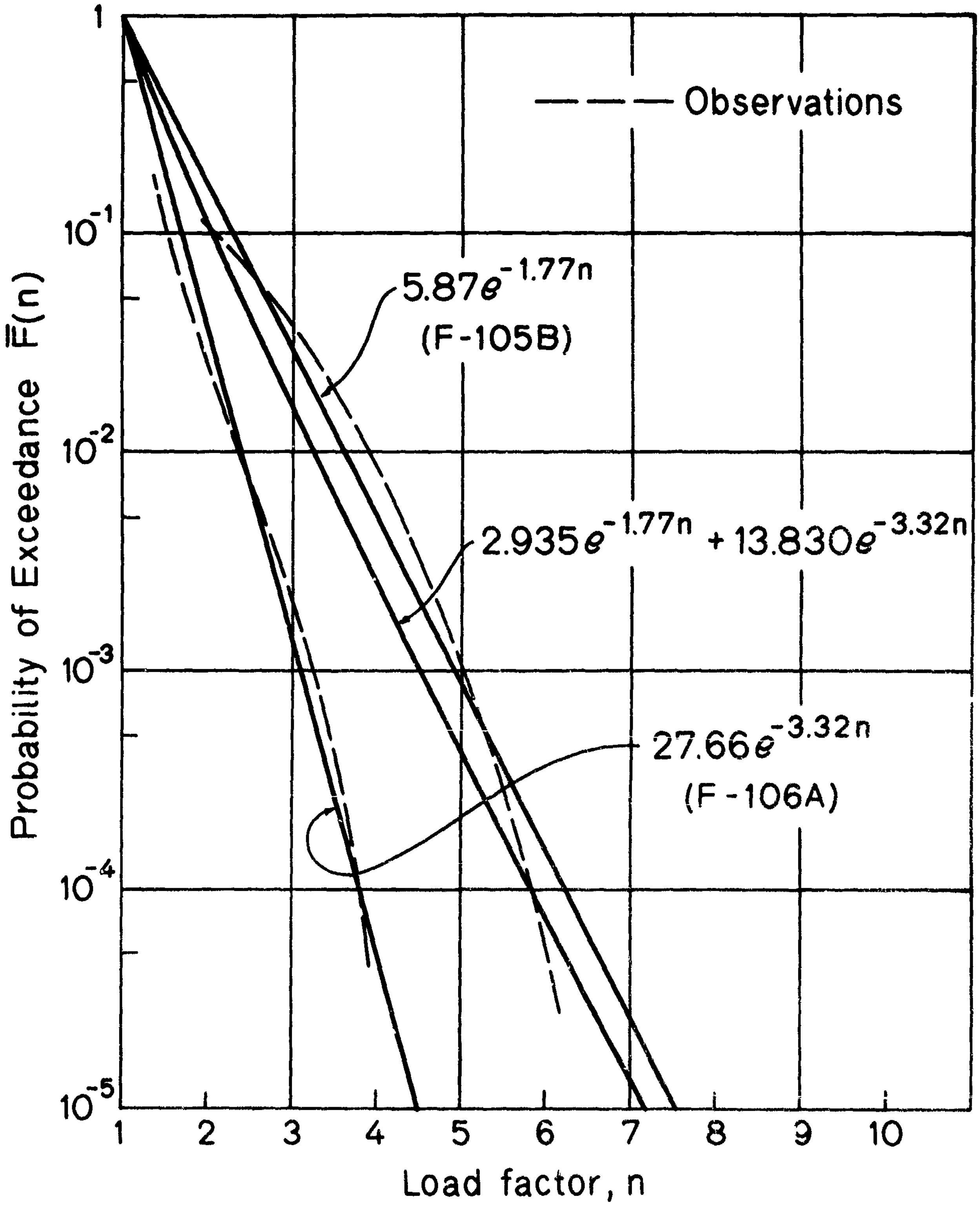

Fig. 13 Probability distributions of flight (manuever) load factor.

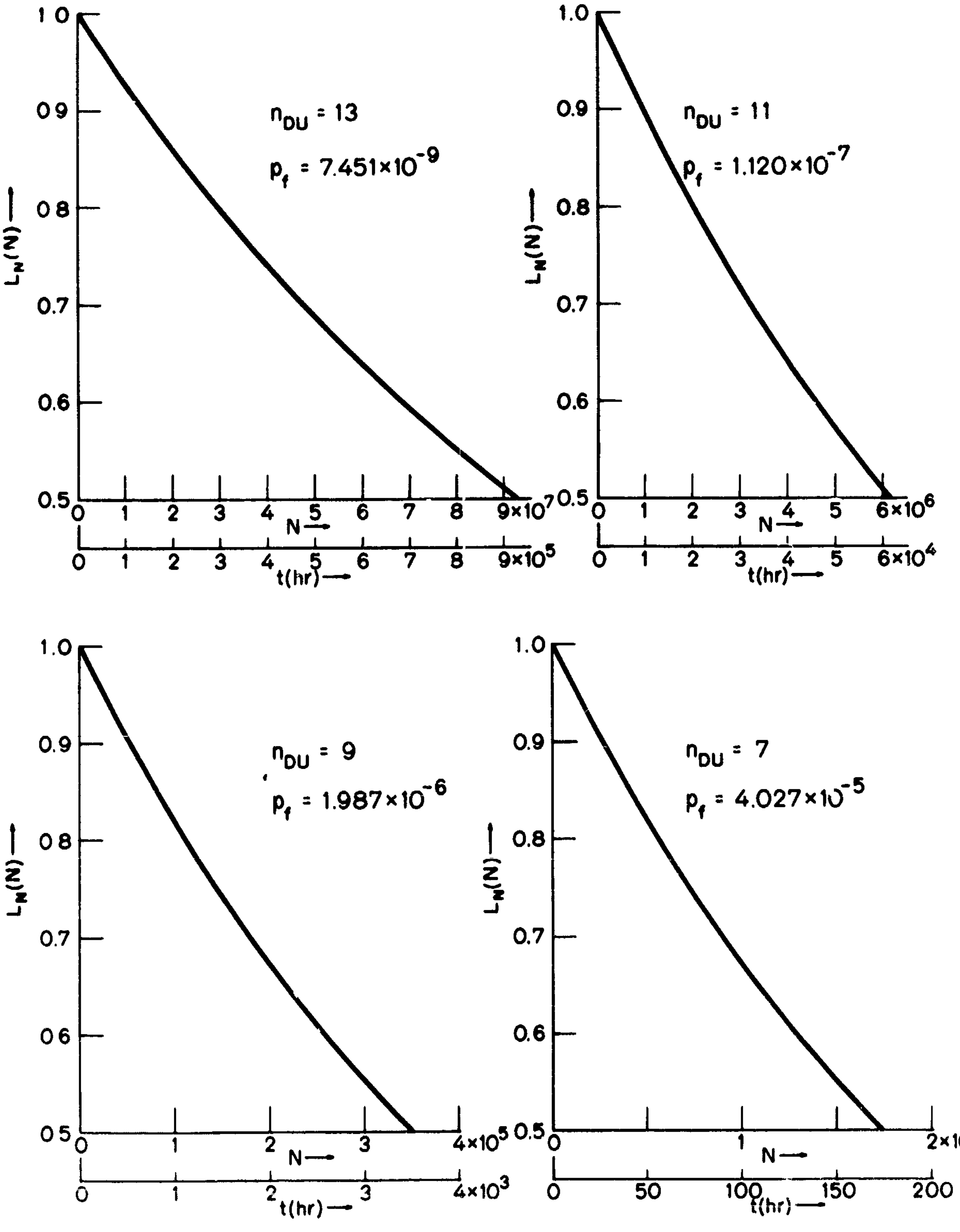

Fig. 14 Probability of survival under flight (manuever) load factors.

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13. ABSTRACT The results of ultimate static strength tests from different types of aircraft structures and structural parts obtained from several aircraft manufacturers were statistically analyzed. By using test samples with at least 3 replications and reducing sample data to their mean, all results could be unified in a single population of over 300 data points and these points fitted by the Third Asymptotic distribution of smallest values (Weibull distribution). This distribution is used as a representative distribution of the ultimate strength of an aircraft combined with the ratio between the design ultimate load and the ultimate strength attained in actual tests, derived from the test data.

By combining the distribution of strength with representative distributions of gusts in flight through thunderstorm turbulence and of operational loads respectively, realistic reliability functions for ultimate load failure of gust-sensitive (long range) and of maneuver sensitive (short range) aircraft structures were obtained for various assumed levels of the ultimate design load.

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